CS 210 Discrete Mathematics
Instructor: Djamel Bouchaffra
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Homework #2

1. Describe an algorithm that takes a list of n integers ($n \geq 1$) and finds the location of the last even integer in the list, or returns 0 if there are no even integers in the list.

2. Show how the binary search algorithm searches for 27 in the following list:

5 7 9 15 16 21 25 30.

3. You have supplies of boards that are one foot, five feet, seven feet, and twelve feet long. You need to lay pieces end-to-end to make a molding 15 feet long and wish to do this using the fewest number of pieces possible. Explain why the greedy algorithm of taking boards of the longest length at each stage (so long as the total length of the boards selected does not exceed 15 feet) does not give the fewest number of boards possible.

4. Let $f(n) = 3n^2 + 15n + 12$. Show that $f(n)$ is $O(n^2)$. Find C and k from the definition.

In the questions below find the best big-oh function for the function. Choose your answer from among the following: 1, $\log_2 n$, n, $n \log_2 n$, n^2, n^3,..., 2^n, $n!$.

5. $f(n) = 1 + 4 + 7 + ... + (3n + 1)$.

6. $g(n) = 1 + 3 + 5 + 7 + ... + (2n - 1)$.

7. $\dfrac{3 - 2n^4 - 4n}{2n^3 - 3n}$.

8. Show that $\sum_{j=1}^{n} (j^3 + j)$ is $O(n^4)$.

9. Find the best big-O function for $n^3 + \sin n^7$.

10. Find the best big-O function for $\dfrac{x^3 + 7x}{3x + 1}$.

In the questions below find the best big-oh function for the function. Choose your answer from among the following: 1, $\log_2 n$, n, $n \log_2 n$, n^2, n^3,..., 2^n, $n!$.

11. An algorithm that lists all ways to put the numbers 1, 2, 3,…,n in a row.

12. The number of print statements in the following:

```
i := 1, j := 1
while i ≤ n
 begin
  while j ≤ i
   begin
    print ``hello";
   j := j + 1
    end
  i := i + 1
  end.
```

13. The number of print statements in the following:

```
while n > 1
 begin
 print ``hello";
 n := ⌊n/2⌋
 end.
```

14. Prove or disprove: For all integers a,b,c,d, if $a \mid b$ and $c \mid d$, then $(a + c)|(b + d)$.

15. Find gcd(20!,12!) by directly finding the largest divisor of both numbers.

16. Suppose that the lcm of two numbers is 400 and their gcd is 10. If one of the numbers is 50, find the other number.

17. Find the hexadecimal expansion of $\mathrm{ABC}_{16} + \mathrm{2F5}_{16}$.

18. What sequence of pseudorandom numbers is generated using the pure multiplicative generator $x_{n+1} = 3x_n$ **mod**11 with seed $x_0 = 2$?

19. Encrypt the message NEED HELP by translating the letters into numbers, applying the encryption function $f(p) = (p + 3)$ **mod** 26, and then translating the numbers back into letters.

20. Suppose that a computer has only the memory locations 0,1,2,...,19. Use the hashing function h where $h(x) = (x + 5)$ **mod** 20 to determine the memory locations in which 57, 32, and 97 are stored.

21. A message has been encrypted using the function $f(x) = (x + 5)$ **mod** 26. If the message in coded form is JCFHY, decode the message.

22. Solve the linear congruence $5x \equiv 3 \pmod{11}$.

23. Use the Principle of Mathematical Induction to prove that $1 + 2^n \leq 3^n$ for all $n \geq 1$.

24. Use the Principle of Mathematical Induction to prove that $1+4+7+10+...+(3n-2) = \dfrac{n(3n-1)}{2}$ for all $n \geq 1$.