Preparation for the Final Exam

1. Find a proposition with three variables p, q, and r that is true when p and r are true and q is false, and false otherwise

2. Find a proposition using only $p,q,\neg$ and the connective $\vee$ with the given truth table.

p	q	?
T	T	F
T	F	T
F	T	T
F	F	F

3. Determine whether this proposition is a tautology: $((p \to \neg q) \wedge q) \to \neg p$.

Use the following to answer questions 4-6:

In the questions below write the statement in the form "If ..., then ...."

4. x is even only if y is odd.

5. A implies B.

6. It is hot whenever it is sunny.

7. Find three subsets of {1,2,3,4,5,6,7,8,9} such that the intersection of any two has size 2 and the intersection of all three has size 1.

8. Suppose U = {1, 2, ..., 9}, A = all multiples of 2, B = all multiples of 3, and C = {3, 4, 5, 6, 7}. Find C - (B - A).

9. Describe an algorithm that takes a list of n integers ($n \geq 1$) and finds the average of the largest and smallest integers in the list.

10. Use the definition of big-oh to prove that $1^2 + 2^2 + ... + n^2$ is $O(n^3)$.

11. Use the definition of big-oh to prove that $\dfrac{3n-8-4n^3}{2n-1}$ is $O(n^2)$.

12. Use the Principle of Mathematical Induction to prove that

$$1-2+2^2-2^3+...+(-1)^n 2^n = \frac{2^{n+1}(-1)^n+1}{3}$$ for all positive integers n.

13. Use the Principle of Mathematical Induction to prove that $1 + 2^n \leq 3^n$ for all $n \geq 1$.

14. Three coins are tossed.
    (a) List the elements in the sample space.
    (b) Find the probability that exactly two heads show.

15. Suppose you and a friend each choose at random an integer between 1 and 8. For example, some possibilities are (3,7), (7,3), (4,4), (8,1), where your number is written first and your friend's number second. Find
    (a) p(you pick 5 and your friend picks 8).
    (b) p(sum of the two numbers picked is < 4).
    (c) p(both numbers match).
    (d) p(the sum of the two numbers is a prime).
    (e) p(your number is greater than your friend's number).

In the questions below suppose you have a class with 30 students — 10 freshmen, 12 sophomores, and 8 juniors.

16. You pick one student at random. What is the probability that the student is not a junior?

17. You pick two students at random, one at a time. What is the probability that both are freshmen?

18. You pick two students at random, one at a time. What is the probability that the second student is a freshman, given that the first is a freshman?

19. In a certain lottery game, three distinct numbers between 10 and 25 (inclusive) are chosen as the winning numbers. What is the probability that the winning numbers are all composite numbers.