

8. Find a proposition with three variables p , q , and r that is true when p and r are true and q is false, and false otherwise

Ans: (a) $p \wedge \neg q \wedge r$.

9. Find a proposition with three variables p , q , and r that is true when exactly one of the three variables is true, and false otherwise

Ans: $(p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$.

10. Find a proposition with three variables p , q , and r that is never true

Ans: $(p \wedge \neg p) \vee (q \wedge \neg q) \vee (r \wedge \neg r)$.

11. Find a proposition using only p, q, \neg and the connective \vee with the given truth table.

p	q	?
T	T	F
T	F	T
F	T	T
F	F	F

Ans: $\neg(\neg p \vee q) \vee \neg(p \vee \neg q)$.

12. Determine whether $p \rightarrow (q \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are equivalent.

Ans: Not equivalent. Let q be false and p and r be true.

13. Determine whether $p \rightarrow (q \rightarrow r)$ is equivalent to $(p \rightarrow q) \rightarrow r$.

Ans: Not equivalent. Let p , q , and r be false.

14. Determine whether $(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q$.

Ans: Both truth tables are identical:

p	q	$(p \rightarrow q) \wedge (\neg p \rightarrow q)$	q
T	T	T	T
T	F	F	F
F	T	T	T
F	F	F	F

15. Write a proposition equivalent to $p \vee \neg q$ that uses only p, q, \neg and the connective \wedge .

Ans: $\neg(\neg p \wedge q)$.

16. Write a proposition equivalent to $\neg p \wedge \neg q$ using only p, q, \neg and the connective \vee .

Ans: $\neg(p \vee q)$.

In the questions below write the statement in the form “If ..., then”

27. x is even only if y is odd.

Ans: If x is even, then y is odd.

28. A implies B .

Ans: If A , then B .

29. It is hot whenever it is sunny.

Ans: If it is sunny, then it is hot.

30. To get a good grade it is necessary that you study.

Ans: If you don't study, then you don't get a good grade (equivalently, if you get a good grade, then you study).

31. Studying is sufficient for passing.

Ans: If you study, then you pass.

32. The team wins if the quarterback can pass.

Ans: If the quarterback can pass, then the team wins.

33. You need to be registered in order to check out library books.

Ans: If you are not registered, then you cannot check out library books (equivalently, if you check out library books, then you are registered).

34. Write the contrapositive, converse, and inverse of the following: If you try hard, then you will win.

Ans: Contrapositive: If you will not win, then you do not try hard. Converse: If you will win, then you try hard. Inverse: If you do not try hard, then you will not win.

35. Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.

Ans: Contrapositive: If you do not sleep late, then it is not Saturday. Converse: If you sleep late, then it is Saturday. Inverse: If it is not Saturday, then you do not sleep late.

Use the following to answer questions 36-38:

In the questions below write the negation of the statement. (Don't write “It is not true that”)

36. It is Thursday and it is cold.

Ans: It is not Thursday or it is not cold.

37. I will go to the play or read a book, but not both.
 Ans: I will go to the play and read a book, or I will not go to the play and not read a book.
38. If it is rainy, then we go to the movies.
 Ans: It is rainy and we do not go to the movies.
39. Explain why the negation of "Al and Bill are absent" is not "Al and Bill are present".
 Ans: Both propositions can be false at the same time. For example, Al could be present and Bill absent.
40. Using c for "it is cold" and d for "it is dry", write "It is neither cold nor dry" in symbols.
 Ans: $\neg c \wedge \neg d$.
41. Using c for "it is cold" and r for "it is rainy", write "It is rainy if it is not cold" in symbols.
 Ans: $\neg c \rightarrow r$.
42. Using c for "it is cold" and w for "it is windy", write "To be windy it is necessary that it be cold" in symbols.
 Ans: $w \rightarrow c$.
43. Using c for "it is cold", r for "it is rainy", and w for "it is windy", write "It is rainy only if it is windy and cold" in symbols.
 Ans: $r \rightarrow (w \wedge c)$.
44. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?
 The system is in multiuser state if and only if it is operating normally.
 If the system is operating normally, the kernel is functioning.
 The kernel is not functioning or the system is in interrupt mode.
 If the system is not in multiuser state, then it is in interrupt mode.
 The system is in interrupt mode.
 Ans: Using m, n, k , and i , there are three rows of the truth table that have all five propositions true: the rows TTTT, FFTT, FFFT for m, n, k, i .
45. On the island of knights and knaves you encounter two people, A and B . Person A says, " B is a knave." Person B says, "We are both knights." Determine whether each person is a knight or a knave.
 Ans: A is a knight, B is a knave.

In the questions below suppose the variable x represents students and y represents courses, and:

$F(x)$: x is a freshman $A(x)$: x is a part-time student $T(x,y)$: x is taking y .

Write the statement in good English without using variables in your answers.

76. $F(\text{Mikko})$.

Ans: Mikko is a freshman.

77. $\neg\exists yT(\text{Joe},y)$.

Ans: Joe is not taking any course.

78. $\exists x(A(x) \wedge \neg F(x))$.

Ans: Some part-time students are not freshmen.

Use the following to answer questions 79-81:

In the questions below suppose the variable x represents students and y represents courses, and:

$M(y)$: y is a math course $F(x)$: x is a freshman

$B(x)$: x is a full-time student $T(x,y)$: x is taking y .

Write the statement in good English without using variables in your answers.

79. $\forall x\exists yT(x,y)$.

Ans: Every student is taking a course.

80. $\exists x\forall yT(x,y)$.

Ans: Some student is taking every course.

81. $\forall x\exists y[(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x,y))]$.

Ans: Every full-time freshman is taking a math course.

Use the following to answer questions 82-84:

In the questions below suppose the variables x and y represent real numbers, and

$L(x,y)$: $x < y$ $G(x)$: $x > 0$ $P(x)$: x is a prime number.

Write the statement in good English without using any variables in your answer.

82. $L(7,3)$.

Ans: $7 < 3$.

83. $\forall x\exists yL(x,y)$.

Ans: There is no largest number.

84. $\forall x\exists y[G(x) \rightarrow (P(y) \wedge L(x,y))]$.

Ans: No matter what positive number is chosen, there is a larger prime.

Use the following to answer questions 85-87:

In the questions below suppose the variables x and y represent real numbers, and

$L(x,y) : x < y$ $Q(x,y) : x = y$ $E(x) : x$ is even $I(x) : x$ is an integer.

Write the statement using these predicates and any needed quantifiers.

85. Every integer is even.

Ans: $\forall x(I(x) \rightarrow E(x))$.

86. If $x < y$, then x is not equal to y .

Ans: $\forall x \forall y(L(x,y) \rightarrow \neg Q(x,y))$.

87. There is no largest real number.

Ans: $\forall x \exists y L(x,y)$.

Use the following to answer questions 88-89:

In the questions below suppose the variables x and y represent real numbers, and

$E(x) : x$ is even $G(x) : x > 0$ $I(x) : x$ is an integer.

Write the statement using these predicates and any needed quantifiers.

88. Some real numbers are not positive.

Ans: $\exists x \neg G(x)$.

89. No even integers are odd.

Ans: $\neg \exists x(I(x) \wedge E(x) \wedge \neg E(x))$.

Use the following to answer questions 90-92:

In the questions below suppose the variable x represents people, and

$F(x) : x$ is friendly $T(x) : x$ is tall $A(x) : x$ is angry.

Write the statement using these predicates and any needed quantifiers.

90. Some people are not angry.

Ans: $\exists x \neg A(x)$.

91. All tall people are friendly.

Ans: $\forall x(T(x) \rightarrow F(x))$.

92. No friendly people are angry.

Ans: $\forall x(F(x) \rightarrow \neg A(x))$.