

8. Find a proposition with three variables  $p$ ,  $q$ , and  $r$  that is true when  $p$  and  $r$  are true and  $q$  is false, and false otherwise
9. Find a proposition with three variables  $p$ ,  $q$ , and  $r$  that is true when exactly one of the three variables is true, and false otherwise
10. Find a proposition with three variables  $p$ ,  $q$ , and  $r$  that is never true
11. Find a proposition using only  $p, q, \neg$  and the connective  $\vee$  with the given truth table.

$p$	$q$	$?$
T	T	F
T	F	T
F	T	T
F	F	F

12. Determine whether  $p \rightarrow (q \rightarrow r)$  and  $p \rightarrow (q \wedge r)$  are equivalent.
13. Determine whether  $p \rightarrow (q \rightarrow r)$  is equivalent to  $(p \rightarrow q) \rightarrow r$ .
14. Determine whether  $(p \rightarrow q) \wedge (\neg p \rightarrow q) \equiv q$ .
15. Write a proposition equivalent to  $p \vee \neg q$  that uses only  $p, q, \neg$  and the connective  $\wedge$ .
16. Write a proposition equivalent to  $\neg p \wedge \neg q$  using only  $p, q, \neg$  and the connective  $\vee$ .

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In the questions below write the statement in the form "If ..., then ...."

27.  $x$  is even only if  $y$  is odd.
28.  $A$  implies  $B$ .
29. It is hot whenever it is sunny.
30. To get a good grade it is necessary that you study.
31. Studying is sufficient for passing.

32. The team wins if the quarterback can pass.
33. You need to be registered in order to check out library books.
34. Write the contrapositive, converse, and inverse of the following: If you try hard, then you will win.
35. Write the contrapositive, converse, and inverse of the following: You sleep late if it is Saturday.

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Use the following to answer questions 36-38:

In the questions below write the negation of the statement. (Don't write "It is not true that ...")

36. It is Thursday and it is cold.
37. I will go to the play or read a book, but not both.
38. If it is rainy, then we go to the movies.
39. Explain why the negation of "Al and Bill are absent" is not "Al and Bill are present".
40. Using  $c$  for "it is cold" and  $d$  for "it is dry", write "It is neither cold nor dry" in symbols.
41. Using  $c$  for "it is cold" and  $r$  for "it is rainy", write "It is rainy if it is not cold" in symbols.
42. Using  $c$  for "it is cold" and  $w$  for "it is windy", write "To be windy it is necessary that it be cold" in symbols.
43. Using  $c$  for "it is cold",  $r$  for "it is rainy", and  $w$  for "it is windy", write "It is rainy only if it is windy and cold" in symbols.
44. A set of propositions is *consistent* if there is an assignment of truth values to each of the variables in the propositions that makes each proposition true. Is the following set of propositions consistent?  
The system is in multiuser state if and only if it is operating normally.  
If the system is operating normally, the kernel is functioning.  
The kernel is not functioning or the system is in interrupt mode.  
If the system is not in multiuser state, then it is in interrupt mode.  
The system is in interrupt mode.
45. On the island of knights and knaves you encounter two people,  $A$  and  $B$ . Person  $A$  says, " $B$  is a knave." Person  $B$  says, "We are both knights." Determine whether each person is a knight or a knave.

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In the questions below suppose the variable  $x$  represents students and  $y$  represents courses, and:

$F(x)$ :  $x$  is a freshman    $A(x)$ :  $x$  is a part-time student    $T(x,y)$ :  $x$  is taking  $y$ .

Write the statement in good English without using variables in your answers.

76.  $F(\text{Mikko})$ .

77.  $\neg\exists yT(\text{Joe},y)$ .

78.  $\exists x(A(x) \wedge \neg F(x))$ .

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Use the following to answer questions 79-81:

In the questions below suppose the variable  $x$  represents students and  $y$  represents courses, and:

$M(y)$ :  $y$  is a math course    $F(x)$ :  $x$  is a freshman

$B(x)$ :  $x$  is a full-time student    $T(x,y)$ :  $x$  is taking  $y$ .

Write the statement in good English without using variables in your answers.

79.  $\forall x\exists yT(x,y)$ .

80.  $\exists x\forall yT(x,y)$ .

81.  $\forall x\exists y[(B(x) \wedge F(x)) \rightarrow (M(y) \wedge T(x,y))]$ .

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Use the following to answer questions 82-84:

In the questions below suppose the variables  $x$  and  $y$  represent real numbers, and

$L(x,y)$ :  $x < y$     $G(x)$ :  $x > 0$     $P(x)$ :  $x$  is a prime number.

Write the statement in good English without using any variables in your answer.

82.  $L(7,3)$ .

83.  $\forall x\exists yL(x,y)$ .

84.  $\forall x\exists y[G(x) \rightarrow (P(y) \wedge L(x,y))]$ .

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Use the following to answer questions 85-87:

In the questions below suppose the variables  $x$  and  $y$  represent real numbers, and

$L(x,y)$ :  $x < y$     $Q(x,y)$ :  $x = y$     $E(x)$ :  $x$  is even    $I(x)$ :  $x$  is an integer.

Write the statement using these predicates and any needed quantifiers.

85. Every integer is even.

86. If  $x < y$ , then  $x$  is not equal to  $y$ .

87. There is no largest real number.

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Use the following to answer questions 88-89:

In the questions below suppose the variables  $x$  and  $y$  represent real numbers, and

$E(x)$  :  $x$  is even    $G(x)$  :  $x > 0$     $I(x)$  :  $x$  is an integer.

Write the statement using these predicates and any needed quantifiers.

88. Some real numbers are not positive.

89. No even integers are odd.

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Use the following to answer questions 90-92:

In the questions below suppose the variable  $x$  represents people, and

$F(x)$ :  $x$  is friendly    $T(x)$ :  $x$  is tall    $A(x)$ :  $x$  is angry.

Write the statement using these predicates and any needed quantifiers.

90. Some people are not angry.

91. All tall people are friendly.

92. No friendly people are angry.