1. Prove or disprove: $A - (B \cap C) = (A - B) \cup (A - C)$.

2. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

3. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving an element table proof.

4. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a proof using logical equivalence.
$$\begin{aligned}\overline{A \cap B} &= \{x \mid x \in \overline{A \cap B}\} \\ &= \{x \mid x \notin A \cap B\} \\ &= \{x \mid \neg(x \in A \cap B)\} \\ &= \{x \mid \neg(x \in A \wedge x \in B)\}\end{aligned}$$

5. Prove that $\overline{A \cap B} = \overline{A} \cup \overline{B}$ by giving a Venn diagram proof.

6. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a containment proof (that is, prove that the left side is a subset of the right side and that the right side is a subset of the left side).

7. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving an element table proof.

8. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a proof using logical equivalence.
$$\begin{aligned}A \cap (B \cup C) &= \{x \mid x \in A \cap (B \cup C)\} \\ &= \{x \mid x \in A \wedge x \in (B \cup C)\} \\ &= \{x \mid x \in A \wedge (x \in B \vee x \in C)\}\end{aligned}$$

9. Prove that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by giving a Venn diagram proof.

In the questions below determine whether the set is finite or infinite. If the set is finite, find its size.

73. $\{x \mid x \in \mathbf{Z} \text{ and } x^2 < 10\}$.

74. $P(\{a,b,c,d\})$, where P denotes the power set.

75. $\{1,3,5,7,\ldots\}$.

76. $A \times B$, where $A = \{1,2,3,4,5\}$ and $B = \{1,2,3\}$.

77. $\{x \mid x \in \mathbf{N} \text{ and } 9x^2 - 1 = 0\}$.
Ans: 0.

78. $P(A)$, where A is the power set of $\{a,b,c\}$.

79. $A \times B$, where $A = \{a,b,c\}$ and $B = \varnothing$.

80. $\{x \mid x \in \mathbf{N} \text{ and } 4x^2 - 8 = 0\}$.

81. $\{x \mid x \in \mathbf{Z} \text{ and } x^2 = 2\}$.

82. $P(A)$, where $A = P(\{1,2\})$.

83. $\{1,10,100,1000,\ldots\}$.

84. $S \times T$, where $S = \{a,b,c\}$ and $T = \{1,2,3,4,5\}$.

85. $\{x \mid x \in \mathbf{Z} \text{ and } x^2 < 8\}$.

86. Prove that between every two rational numbers a/b and c/d
(a) there is a rational number.
(b) there are an infinite number of rational numbers.

87. Prove that there is no smallest positive rational number.

Use the following to answer questions 88-96:

In the questions below determine whether the rule describes a function with the given domain and codomain.

88. $f : \mathbf{N} \to \mathbf{N}$ where $f(n) = \sqrt{n}$.

89. $h : \mathbf{R} \to \mathbf{R}$ where $h(x) = \sqrt{x}$.

90. $g : \mathbf{N} \to \mathbf{N}$ where $g(n)$ = any integer $> n$.

91. $F : \mathbf{R} \to \mathbf{R}$ where $F(x) = \dfrac{1}{x-5}$.

92. $F : \mathbf{Z} \to \mathbf{R}$ where $F(x) = \dfrac{1}{x^2-5}$.

93. $F : \mathbf{Z} \to \mathbf{Z}$ where $F(x) = \dfrac{1}{x^2-5}$.

94. $G : \mathbf{R} \to \mathbf{R}$ where $G(x) = \begin{cases} x+2 & \text{if } x \geq 0 \\ x-1 & \text{if } x \leq 4 \end{cases}$

95. $f : \mathbf{R} \to \mathbf{R}$ where $f(x) = \begin{cases} x^2 & \text{if } x \leq 2 \\ x-1 & \text{if } x \geq 4 \end{cases}$

96. $G : \mathbf{Q} \to \mathbf{Q}$ where $G(p/q) = q$.

97. Give an example of a function $f : \mathbf{Z} \to \mathbf{Z}$ that is 1-1 and not onto $\mathbf{Z}$.

98. Give an example of a function $f : \mathbf{Z} \to \mathbf{Z}$ that is onto $\mathbf{Z}$ but not 1-1.

99. Give an example of a function $f : \mathbf{Z} \to \mathbf{N}$ that is both 1-1 and onto $\mathbf{N}$.