1. Determine whether $p \to (q \to r)$ and $p \to (q \wedge r)$ are equivalent.
   Ans: Not equivalent. Let *q* be false and *p* and *r* be true.

2. Write a proposition equivalent to $\neg p \wedge \neg q$ using only $p,q,\neg$ and the connective $\vee$.
   Ans: $\neg(p \vee q)$.

3. In the questions below write the statement in the form "If ..., then ...."
   The team wins if the quarterback can pass.
   Ans: If the quarterback can pass, then the team wins.

4. You need to be registered in order to check out library books.
   Ans: If you are not registered, then you cannot check out library books (equivalently, if you check out library books, then you are registered).

5. In the questions below write the negation of the statement. (Don't write "It is not true that ....")
   If it is rainy, then we go to the movies.
   Ans: It is rainy and we do not go to the movies.

6. In the questions below suppose $P(x,y)$ is a predicate and the universe for the variables x and y is {1,2,3}. Suppose $P(1,3)$, $P(2,1)$, $P(2,2)$, $P(2,3)$, $P(2,3)$, $P(3,1)$, $P(3,2)$ are true, and $P(x,y)$ is false otherwise. Determine whether the following statements are true.

   $\forall x \exists y P(x,y)$.
   Ans: True

7. In the questions below suppose the variable x represents students and the variable y represents courses, and
   $A(y)$: y is an advanced course $S(x)$: x is a sophomore
   $F(x)$: x is a freshman $T(x,y)$: x is taking y.
8. Write the statement using these predicates and any needed quantifiers.

   There is a course that every freshman is taking.
   Ans: $\exists y \forall x(F(x) \to T(x,y))$.

   No freshman is a sophomore.
   Ans: $\neg\exists x(F(x) \wedge S(x)]$.

9. In the questions below suppose $g : A \to B$ and $f : B \to C$ where $A = B = C = \{1,2,3,4\}$, $g = \{(1,4),(2,1),(3,1),(4,2)\}$ and $f = \{(1,3),(2,2),(3,4),(4,2)\}$.
Find $f \circ g$.
Ans: {(1,2),(2,3),(3,3),(4,2)}.

10. In the questions below suppose $g : A \to B$ and $f : B \to C$ where $A = \{1,2,3,4\}$, $B = \{a,b,c\}$, $C = \{2,8,10\}$, and g and f are defined by $g = \{(1,b),(2,a),(3,b),(4,a)\}$ and $f = \{(a,8),(b,10),(c,2)\}$.

Explain why g^{-1} is not a function.
Ans: $g^{-1}(a)$ **is equal to both 2 and 4.**

11. Describe an algorithm that takes a list of n integers ($n \geq 1$) and finds the location of the last even integer in the list, or returns 0 if there are no even integers in the list.
Ans: procedure *lasteven*($a_1,\ldots,a_n$: integers)
location : = 0
for i : = 1 **to** n
if 2 | a_i **then** *location*: = i.

12. Show how the binary search algorithm searches for 27 in the following list:
5 6 8 12 15 21 25 31.
Ans: The consecutive choices of sublists of the original list are: 15 21 25 31, 25 31, and 25. Since 27 ≠ 25, the integer 25 is not in the list.

13. Use the definition of big-oh to prove that $\dfrac{6n+4n^5-4}{7n^2-3}$ is $O(n^3)$.

Ans: $\dfrac{6n+4n^5-4}{7n^2-3} \leq \dfrac{6n^5+4n^5}{7n^2-n^2} = \dfrac{10n^5}{6n^2} = \dfrac{5}{3}n^3$, **if** $n \geq 2$.

14. Suppose you wish to prove that the following is true for all positive integers n by using the Principle of Mathematical Induction: $1 + 3 + 5 + \ldots + (2n - 1) = n^2$.
(a) Write $P(1)$
(b) Write $P(72)$
(c) Write $P(73)$
(d) Use $P(72)$ to prove $P(73)$
(e) Write $P(k)$
(f) Write $P(k + 1)$
(g) Use the Principle of Mathematical Induction to prove that $P(n)$ is true for all positive integers n

Ans: **(a)** $1 = 1^2$.
(b) $1 + 3 + 5 + \ldots + 143 = 72^2$.
(c) $1 + 3 + 5 + \ldots + 145 = 73^2$.
(d) $1 + 3 + 5 + \ldots + 145 = (1 + 3 + 5 + \ldots + 143) + 145 = 72^2 + 145 = 72^2 + 2 \cdot 72 + 1 = (72 + 1)^2 = 73^2$.
(e) $1 + 3 + \ldots + (2k - 1) = k^2$.

(f) $1 + 3 + ... + (2k + 1) = (k + 1)^2$.
(g) $P(1)$ is true since $1 = 1^2$. $P(k) \rightarrow P(k + 1)$: $1 + 3 + ... + (2k + 1) = k^2 + (2k + 1)$ $= (k + 1)^2$.

15. Find the error in the following proof of this "theorem":
"Theorem: Every positive integer equals the next largest positive integer."
" *Proof*: Let $P(n)$ be the proposition '$n = n + 1$'. To show that $P(k) \rightarrow P(k + 1)$, assume that $P(k)$ is true for some k, so that $k = k + 1$. Add 1 to both sides of this equation to obtain $k + 1 = k + 2$, which is $P(k + 1)$. Therefore $P(k) \rightarrow P(k + 1)$ is true. Hence $P(n)$ is true for all positive integers n."
Ans: No basis case has been shown.